1) 

Given: $\frac{dV}{dt} = 16\ ft^3/min$

$h = \frac{1}{4} D = \frac{2}{4} r = \frac{1}{2} r$

$h = \frac{1}{2} r$

$r = 2h$

SAND is pouring from a pipe at a rate of $16\ ft^3/sec$. The falling sand forms a conical pile on the ground. The altitude of the pile is always $\frac{1}{4}$ the diameter of the base. What is the rate of change of the altitude at the instant that the altitude is 4 ft.

$\frac{dh}{dt}$ when $h = 4ft$?

$V = \frac{\pi}{3} r^2 h$

$V = \frac{\pi}{3}(2h)^2 h = \frac{\pi}{3} 4h^3$

$\frac{dV}{dt} = \frac{\pi}{\cancel{3}} 4 \cdot \cancel{3} h^2 \frac{dh}{dt}$

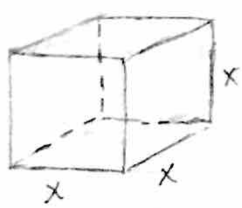
$\frac{dV}{dt} = 4\pi h^2 \frac{dh}{dt}$

$16 = 4\pi h^2 \frac{dh}{dt}$

$\frac{dh}{dt} = \frac{16}{4\pi(16)}$

$\frac{dh}{dt} = \frac{1}{4\pi}\ ft/sec$

2) The edge of a cube is expanding at a rate of 2.5 cm/sec. How fast is the Volume changing when the length of an edge is 15 cm?



$\frac{dx}{dt} = 2.5\ cm/sec$

$\frac{dV}{dt}$ when length of an edge is 15 cm

$V = x^3$

$\frac{dV}{dt} = 3x^2 \frac{dx}{dt}$

$= 3x^2(2.5)$

$= 3(15)^2(2.5)$

$\frac{dV}{dt} = 1687.50\ cm^3$

How fast is the surface area changing at the same instant?

$S = 6x^2$

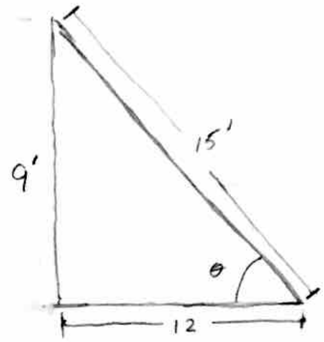
$\frac{dS}{dt} = 12x\left(\frac{dx}{dt}\right)$

$= 12(15)(2.5)$

$\frac{dS}{dt} = 450\ cm^2/sec$

3) A 15-foot ladder leans against a vertical wall. If the top of the ladder slides down the wall at a rate of 2 ft/sec, how fast is the bottom of the ladder moving when it is 12 ft from the wall?

$\frac{dy}{dt} = 2\,\text{ft/sec} \downarrow$



$$\frac{dx}{dt} = ?$$

$$\frac{dy}{dt} = -2\ \text{ft/sec}$$

$$L^2 = x^2 + y^2$$

$$2L\frac{dL}{dt} = 2x\frac{dx}{dt} + 2y\frac{dy}{dt}$$

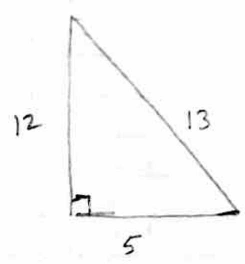
length of ladder does not change

$$30\,(0) = 2(12)\frac{dx}{dt} + 2(9)(2)$$

$$\cdot 24\frac{dx}{dt} = -36$$

$$\frac{dx}{dt} = \frac{-36}{-24} = 1.5\ \text{ft/sec.}$$

4) At a given instant, the legs of a right triangle are 5 cm and 12 cm long. If the short leg is increasing at a rate of 1 cm per second and the long leg is decreasing at a rate of 2 cm per second, how fast is the hypotenuse changing?



$$\frac{dx}{dt} = 1\ \text{cm/sec} \qquad x = 5\ \text{cm}$$

$$\frac{dy}{dt} = -2\ \text{cm/sec} \qquad y = 12$$

$$L = 13$$

$$\frac{dL}{dt} = ?$$

$$L^2 = x^2 + y^2$$

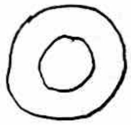
$$2L\frac{dL}{dt} = 2x\frac{dx}{dt} + 2y\cdot\frac{dy}{dt}$$

$$2(13)\frac{dL}{dt} = 2(5)(1) + (2)(12)(-2)$$

$$26\frac{dL}{dt} = 10 - 48 \qquad \frac{dL}{dt} = -\frac{19}{13}\ \text{cm/sec.}$$

$$\frac{dL}{dt} = -\frac{38}{26} = -\frac{19}{13}\ \text{cm/sec}$$

5) A stone is dropped into a still pond and it produces circular ripples. If the radius of one of the ripples increases at a rate of 2 feet per second, how fast is the enclosed area changing when the radius is 12 ft.? If the area is changing at a rate of 24 sq. ft per second, how fast is the radius changing when the area is 130 sq. ft.



$$\frac{dr}{dt} = 2\ ft/sec$$

$$\frac{dA}{dt} \ @\ r = 12\ ft$$

a) $A = \pi r^2$

$$\frac{dA}{dt} = \pi\, 2r \frac{dr}{dt}$$

$$\frac{dA}{dt} = \pi(2)(12)(2)$$

$$\frac{dA}{dt} = 48\pi = 150.80\ ft^2/sec$$

b) $\frac{dA}{dt} = 24\ ft^2/sec$

$$\frac{dr}{dt} = ?\ @\ A = 130\ ft^2$$

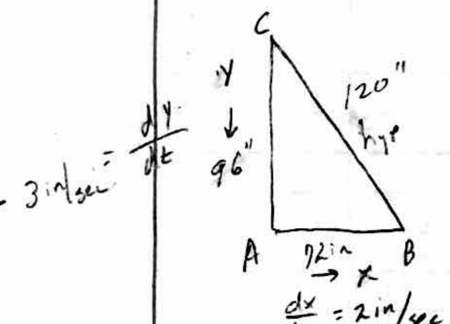
$$24 = 2\pi r \frac{dr}{dt} \Rightarrow 24 = 2\pi(6.433)\frac{dr}{dt}$$

$$A = \pi r^2$$
$$130 = \pi r^2$$
$$r^2 = \frac{130}{\pi}$$
$$r = 6.433\ ft$$

$$\frac{12}{6.433\pi} = \frac{dr}{dt}$$

$$\frac{dr}{dt} = 0.5938\ ft/sec$$

6) If one leg AB of a right triangle increases at the rate of 2 inches per second, while the other leg AC decreases at 3 inches per second, find how fast the hypotenuese is changing when AB = 6 ft and AC = 8 ft



$$h^2 = x^2 + y^2$$

$$2h\frac{dh}{dt} = 2x\frac{dx}{dt} + 2y\frac{dy}{dt}$$

$$2(120)\frac{dh}{dt} = 2(72)(2) + 2(96)(-3)$$

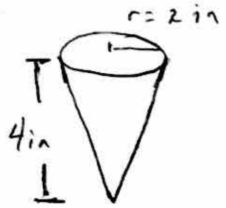
$$240\frac{dh}{dt} \quad 288 + (-576)$$

$$\frac{dh}{dt} = \frac{288}{240} - \frac{576}{240}$$

$$1.2 - 2.4$$

$$\frac{dh}{dt} = -1.2\ in/sec$$

7) The diameter and height of a paper cup in the shape of a cone are both 4 inches and water is leaking out at the rate of $\frac{1}{2}$ in^3/sec. Find the rate at which the water level is dropping when the diameter of the surface is 2 inches.



$\frac{dV}{dt} = -\frac{1}{2}$ in^3/sec

$\frac{dh}{dt} = ?$ when $D = 2$ inch

$r = 1$ inch

$h = 2$ inches

$\frac{r}{h} = \frac{2}{4}$

$r = \frac{1}{2}h$

$V = \frac{\pi}{3}r^2h = \frac{\pi}{3}\left(\frac{1}{2}h\right)^2 h$

$= \frac{\pi}{3}\left(\frac{1}{4}h^2\right)h$

$V = \frac{\pi}{12}h^3$

$\frac{dV}{dt} = \frac{\pi}{12}3h^2\frac{dh}{dt} = \frac{\pi}{4}h^2\frac{dh}{dt}$

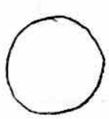
$-\frac{1}{2} = \frac{\pi}{4}(4)\frac{dh}{dt}$

$-\frac{1}{2} = \frac{\pi}{4}(4)\left(\frac{dh}{dt}\right)$

$\frac{-\frac{1}{2}}{\pi} = \frac{dh}{dt}$

$\frac{dh}{dt} = -0.159$ in/sec

8) A balloon is being filled with helium at the rate of 4 ft^3/min. Find the rate, in square ft/min, at which the surface area is increasing when the volume is $\frac{32\pi}{3}$ ft^3



$V = \frac{4}{3}\pi r^3$

$SA = 4\pi r^2$

$\frac{dv}{dt} = 4$ ft^3/min

$\frac{dSA}{dt} = ?$ when $\frac{32\pi}{3}$ ft^3

$\frac{dV}{dt} = \frac{4}{3}\pi\, 3r^2\frac{dr}{dt}$

$\frac{dV}{dt} = 4\pi r^2\frac{dr}{dt}$

$4 = 4\pi r^2\frac{dr}{dt}$

$4 = 4\pi(2)^2\frac{dr}{dt}$

$\frac{1}{4\pi} = \frac{dr}{dt}$

find r

$\frac{32\pi}{3} = \frac{4}{3}\pi r^3$

$\frac{32\pi}{3}\cdot\frac{3}{\pi} = 4r^3$

$32 = 4r^3$

$8 = r^3$

$r = 2$

$\frac{dSA}{dt} = 8\pi r\frac{dr}{dt}$

$\frac{dSA}{dt} = \cancel{8}^{2}\cancel{\pi}(2)\left(\frac{1}{4\cancel{\pi}}\right)$

$= 4$ ft^2/min